

## Extra Practice Problems 6

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Here's yet another set of practice problems. Good luck!

### Equivalent Definitions

In lecture, we initially defined a strict order to be a binary relation that is irreflexive, asymmetric, and transitive. We proved in lecture that a binary relation is a strict order if and only if it's asymmetric and transitive, and in Problem Set Four you proved that a binary relation is a strict order if and only if it's irreflexive and transitive.

Prove or disprove: a binary relation is a strict order if and only if it is irreflexive and asymmetric.

### More Fun With Friends and Strangers

Suppose you have a 17-clique (that is, an undirected graph with 17 nodes where there's an edge between each pair of nodes) where each edge is colored one of *three* different colors (say, red, green, and blue). Prove that regardless of how the 17-clique is colored, it must contain a blue triangle, a red triangle, or a green triangle. (*Hint: Use the theorem on friends and strangers.*)

### Bijections and Induction

Let  $f : \mathbb{N} \rightarrow \mathbb{N}$  be a function. We'll say that  $f$  is *linearly bounded* if  $f(n) \leq n$  for all  $n \in \mathbb{N}$ .

Prove that if  $f : \mathbb{N} \rightarrow \mathbb{N}$  is linearly bounded and is a bijection, then  $f(n) = n$  for all  $n \in \mathbb{N}$ . (*Hint: You might find induction useful here.*)

### DFAs, NFAs, and Regular Expressions

Let  $\Sigma = \{a, b\}$  and consider the following language over  $\Sigma$ :

$$L = \{ w \in \Sigma^* \mid \text{some letter in } w \text{ appears at least four times} \}$$

- i. Design an NFA for  $L$ .
- ii. Write a regular expression for  $L$ .